Course number		U-LAS70 10002 SE50										
Course title (and course title in English)	arithmetic(現代整数論との出会い) Instructor's name, job title, Researc							th Institute for Mathematical Sciences Lecturer,UEDA FUKUHIRO				
Group	Seminar	ars in Liberal Arts and Sciences Number of credits 2					2		Number of weekly time blocks			
Class style semin (Fac		nar e-to-face course	Yea	Year/semesters		2025 • First semeste		r	Quota (Freshman)		5 (15)	
Target year Mainly		y 1st year students	Eligible	studen	ts Fo	or all majors			ys and riods	Thu.5		
Classroom									iguage of truction	English		
Keyword	Algebra	a-related										

[Overview and purpose of the course]

It is a classical question from centuries ago whether a quintic (or of higher degree) polynomial equation is solvable in terms of its coefficients, with only use of the usual operations (addition, subtraction, multiplication, division) and application of radicals (square roots, cube roots, etc). Modern/abstract algebra was born to answer this question, the answer to which turns out to be negative in general. On the other hand, abstract algebra has gone far beyond this and is rightly regarded as one of the central features of modern mathematics nowadays, which is in particular fundamental for the study of arithmetic problems.

[Course objectives]

We will learn the basic concepts and theorems in group theory, ring theory, field theory, and Galois theory. As an application, we shall also be able to determine which polynomial equations are solvable in radicals.

[Course schedule and contents)]

We intend to cover a big chunk of modern algebra in a condensed and interesting way, to make it accessible to most undergraduate students. Both concepts and examples will be emphasized. Below are the plan and contents of the course. The lectures, as well as the order of the lectures, may be modified, depending on students' background and understanding of the course materials.

-Set Theory [1 week]:

Notion of sets, mappings, mathematical induction, Zorn's lemma.

-Group theory [4 weeks]:

Definition and examples of groups, homomorphisms, abelian groups, symmetric groups, Sylow's theorem.

-Ring theory [3 weeks]:

Definition and examples, ideals, quotient rings, Euclidean domains, PIDs, UFDs, polynomial rings.

-Field theory [3 weeks]:

Definition and examples, field extensions, polynomials, finite fields.

-Galois theory [2 weeks]:

Galois extensions, roots of unity, solvability.

-Some applications to arithmetic [1 week]

-Feedback [1 week]

ILAS Seminar-E2: Encounters with modern arithmetic (現代整数論との出会い) (2)
[Course requirements]
None
[Evaluation methods and policy]
The evaluation consists of the following weighted parts:
-Performance in class (20%).
-Presentation (60%): Each student reviews a mathematical topic assigned by the instructor. Such a topic is
typically a section from the textbook below.
-Report (20%): Your report covers the details of your presentation. Each student needs to email the report to the instructor no later than Friday of Week 15.
[Textbooks]
D. Dummit and R. Foote Abstract Algebra (Wiley; 3rd edition) ISBN:9780471433347
There is no need to purchase the textbook. Several pdf versions of this book are available online for free.
[References, etc.]
(References, etc.)
Other supplemental materials, such as handouts from the instructor, may be introduced during the classes.
[Study outside of class (preparation and review)]
Along with preparation and review, students are encouraged to form study groups.
[Other information (office hours, etc.)]
[Essential courses]